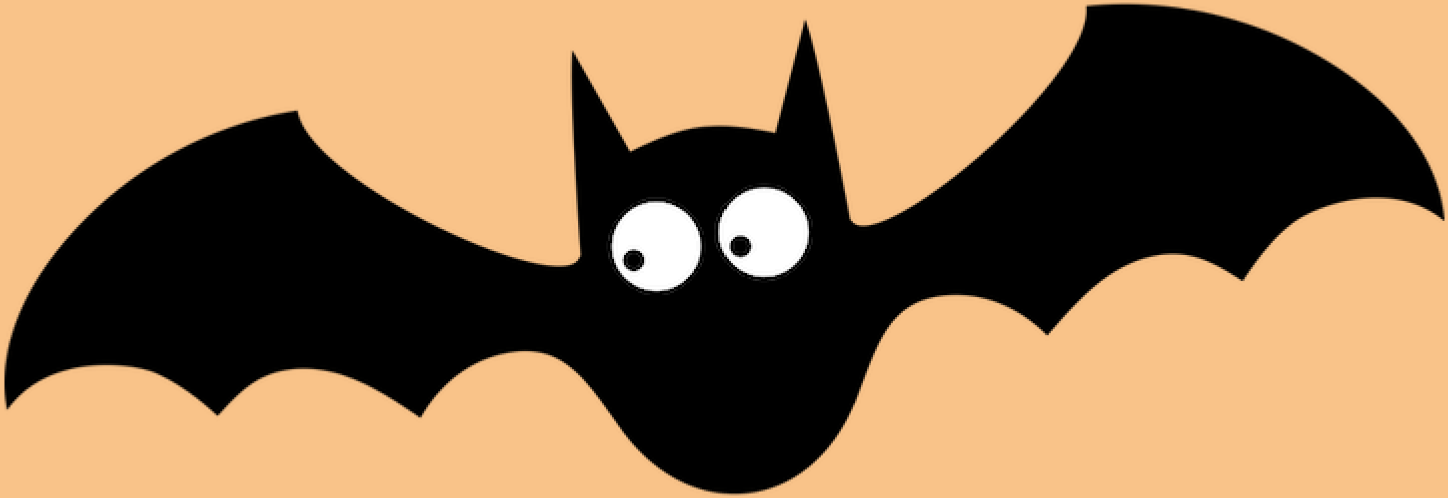


Adaboost

Son Nguyen

Adaboost

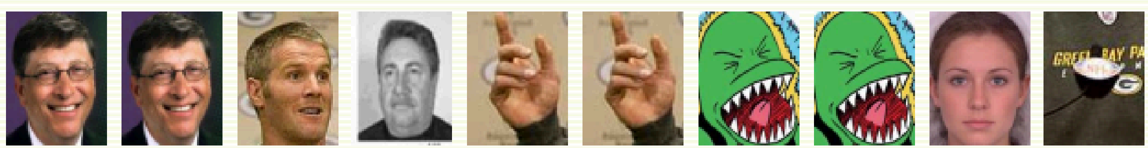
Idea Behind Ada Boost

- Examples of high weight are shown more often at later rounds
- Face/nonface classification problem:

Round 1

	1/7	1/7	1/7	1/7	1/7	1/7	1/7
best weak classifier:	✓	✗	✓	✓	✗	✓	✗
change weights:	1/16	1/4	1/16	1/16	1/4	1/16	1/4

Round 2

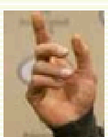


best weak classifier:	✓	✓	✓	✗	✗	✗	✓	✓	✓	✓
change weights:		1/8	1/32	11/32		1/2		1/8	1/32	1/32

Adaboost

Idea Behind Ada Boost

Round 3

- out of all available weak classifiers, we choose the one that works best on the data we have at round 3
- we assume there is always a weak classifier better than random (better than 50% error)
-  image is half of the data given to the classifier
- chosen weak classifier **has to** classify this image correctly

Adaboost, Clearly Explained

- Demonstration by StatQuest
- Link

Calculation Example

Data

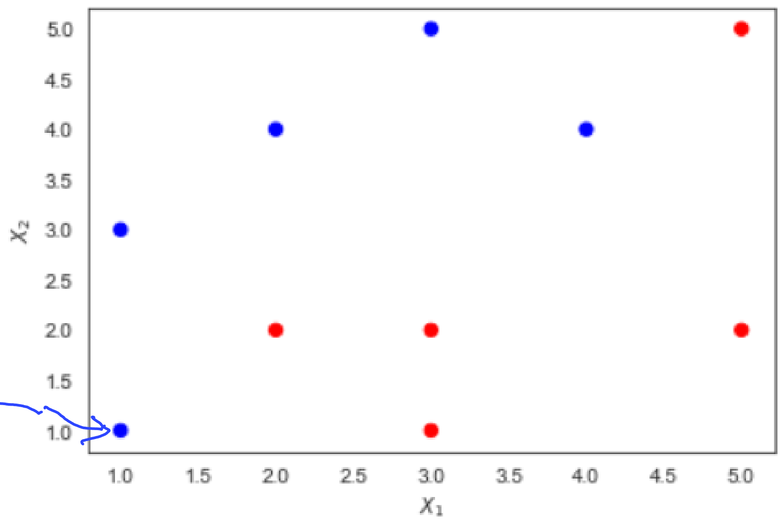
x_1	x_2	**y**
1	1	1
1	3	1
2	2	-1
2	4	1
3	1	-1
3	2	-1
3	5	1
4	4	1
5	2	-1
5	5	-1

Calculation Example

Data

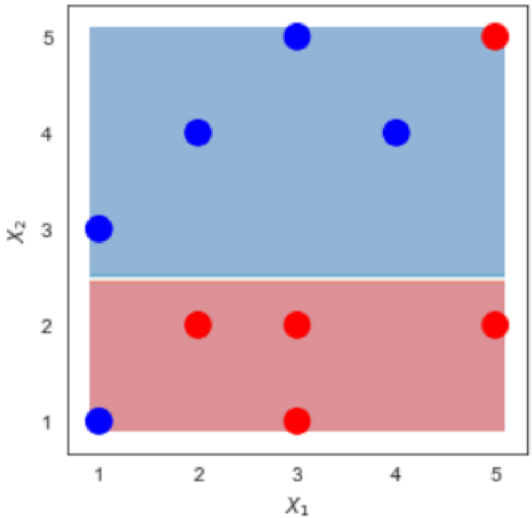
x_1	x_2	**y**
1	1	1
1	3	1
2	2	-1
2	4	1
3	1	-1
3	2	-1
3	5	1
4	4	1
5	2	-1
5	5	-1

blue point

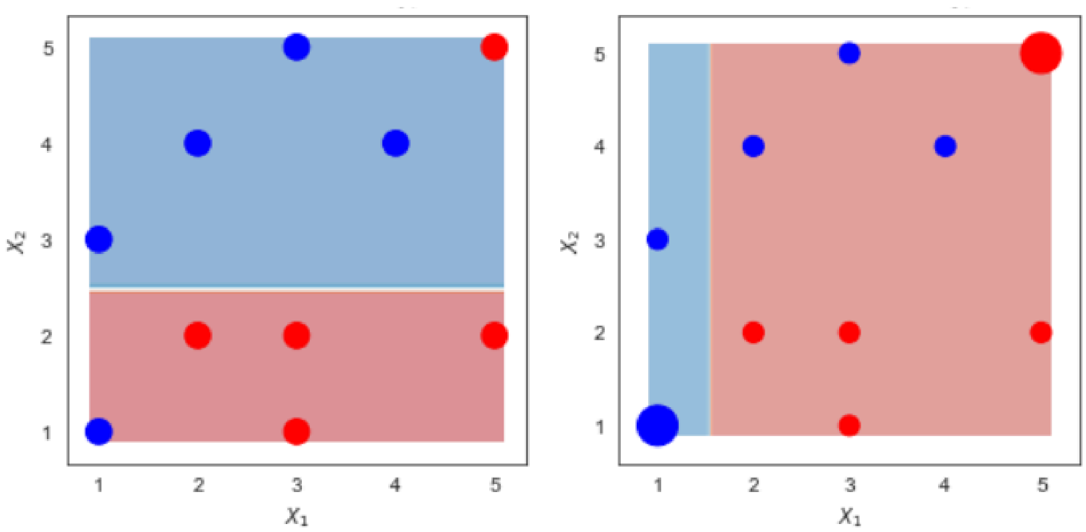


Adaboost in a nutshell

Make Stump 1

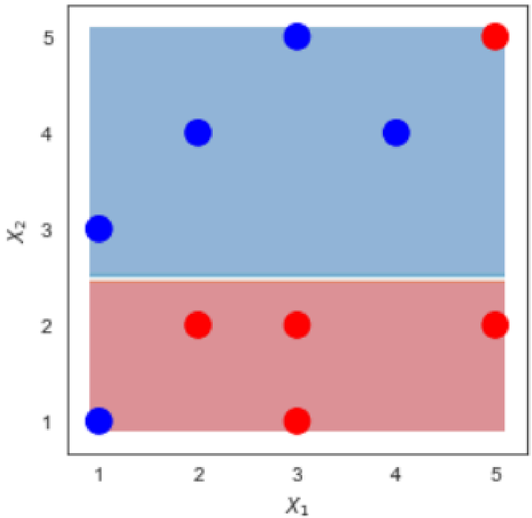


Make Stump 2

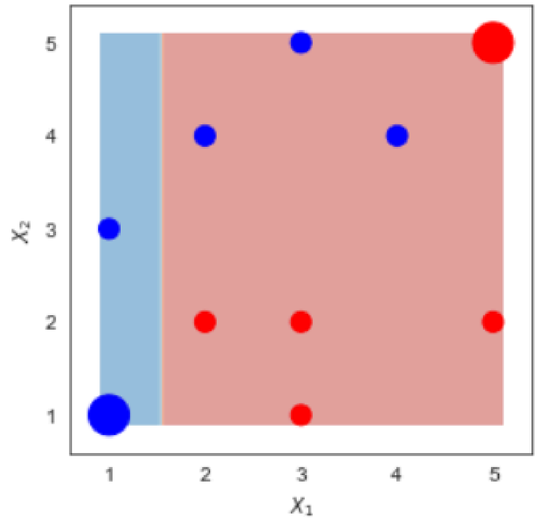


Make Stump 3

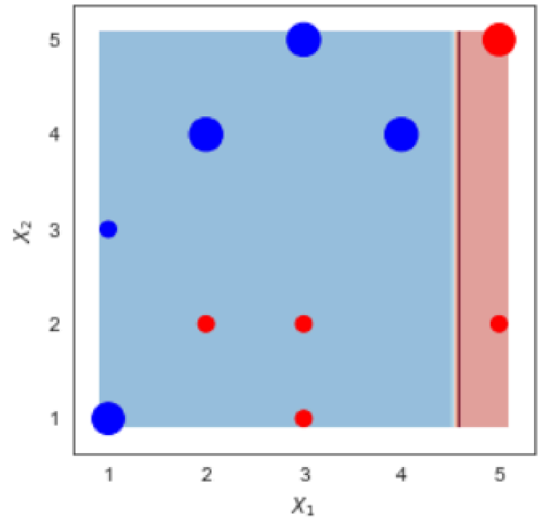
Model 1



Model 2



Model 3

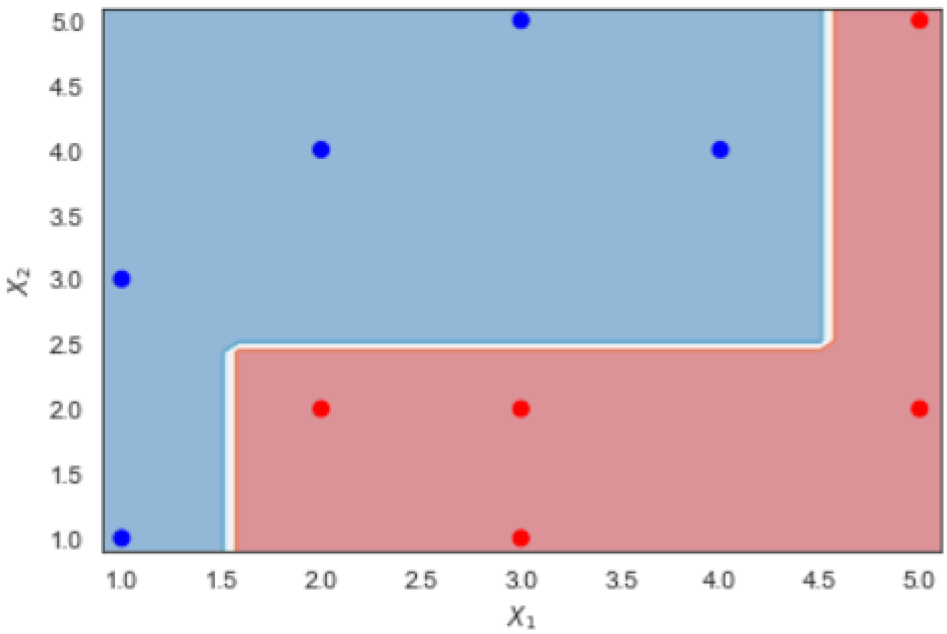


$\alpha_1 = .2$

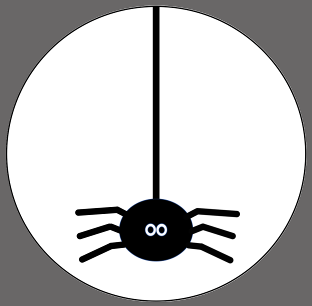
$\alpha_2 =$

$\alpha_3 =$

Combine the Stumps



Detail Calculation



Make the first stump

Row	x1	x2	y
0	1	1	1
1	1	3	1
2	2	2	-1
3	2	4	1
4	3	1	-1
5	3	2	-1
6	3	5	1
7	4	4	1
8	5	2	-1
9	5	5	-1

Make the first stump

- Assign weights for each row
- Every row has the same weight in the first step

Row	x1	x2	y
0	1	1	1
1	1	3	1
2	2	2	-1
3	2	4	1
4	3	1	-1
5	3	2	-1
6	3	5	1
7	4	4	1
8	5	2	-1
9	5	5	-1

Make the first stump

- Assign weights for each row
- Every row has the same weight in the first step

Row	x1	x2	y	Weight 1
0	1	1	1	0.1
1	1	3	1	0.1
2	2	2	-1	0.1
3	2	4	1	0.1
4	3	1	-1	0.1
5	3	2	-1	0.1
6	3	5	1	0.1
7	4	4	1	0.1
8	5	2	-1	0.1
9	5	5	-1	0.1

Make the first stump

- Use Weighted Gini-Index to calculate the children entropy of all candidate splits

Row	x1	x2	y	Weight 1
0	1	1	1	0.1
1	1	3	1	0.1
2	2	2	-1	0.1
3	2	4	1	0.1
4	3	1	-1	0.1
5	3	2	-1	0.1
6	3	5	1	0.1
7	4	4	1	0.1
8	5	2	-1	0.1
9	5	5	-1	0.1

Make the first stump

- Use Weighted Gini-Index to calculate the children entropy of all candidate splits
- The split with the lowest children impurity is the best split

Row	x1	x2	y	Weight 1
0	1	1	1	0.1
1	1	3	1	0.1
2	2	2	-1	0.1
3	2	4	1	0.1
4	3	1	-1	0.1
5	3	2	-1	0.1
6	3	5	1	0.1
7	4	4	1	0.1
8	5	2	-1	0.1
9	5	5	-1	0.1

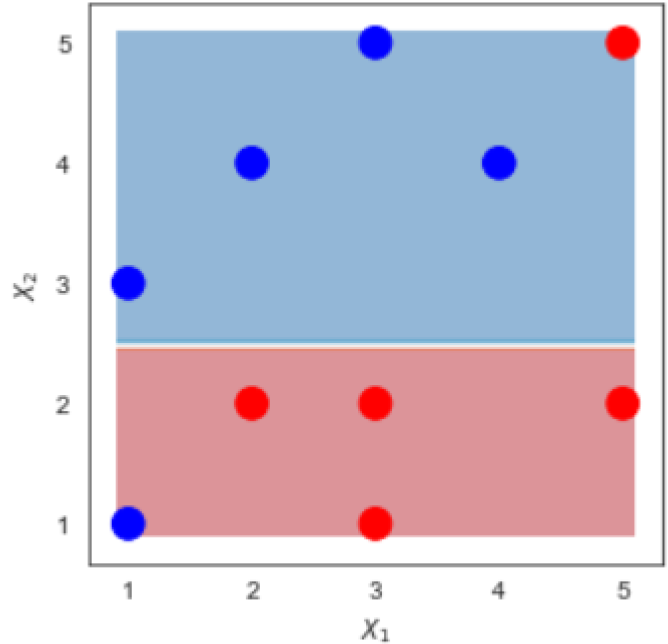
Make the first stump

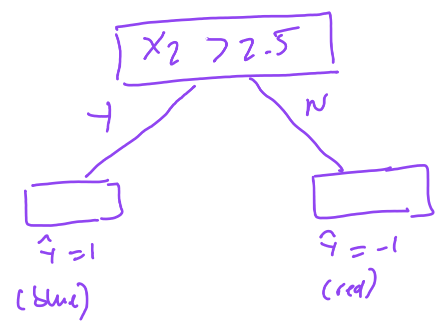
- Use Weighted Gini-Index to calculate the children entropy of all candidate splits
- The split with the lowest children impurity is the best split
- **NOTE**: You are not required to make the stump. So the stump will be given to you!

Row	x1	x2	y	Weight 1
0	1	1	1	0.1
1	1	3	1	0.1
2	2	2	-1	0.1
3	2	4	1	0.1
4	3	1	-1	0.1
5	3	2	-1	0.1
6	3	5	1	0.1
7	4	4	1	0.1
8	5	2	-1	0.1
9	5	5	-1	0.1

Make the first stump

- Use Weighted Gini-Index to calculate the children entropy of all candidate splits
- The split with the lowest children impurity is the best split
- **NOTE**: You are not required to make the stump. So the stump will be given to you!
- Here is the first stump

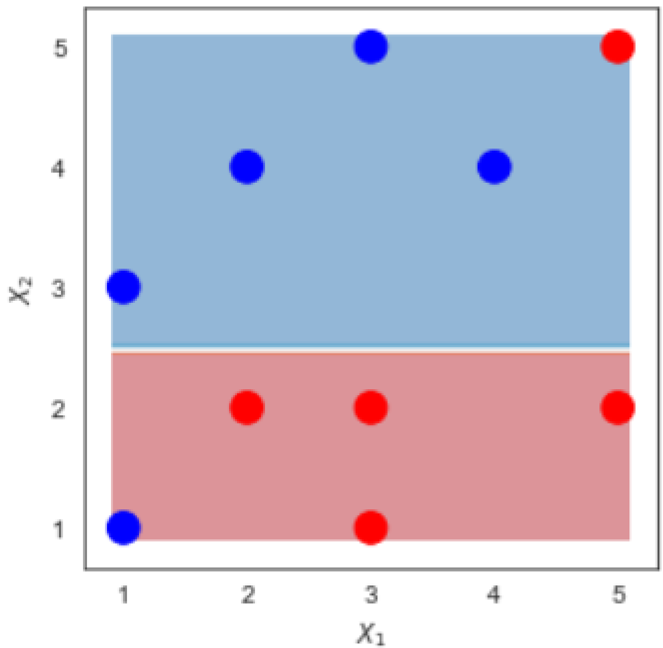




$= I(X_2 > 2.5)$

Make the first stump

- Stump 1: $I(x_2 > 2.5)$



Prediction of Stump 1

- Stump 1:

$$I(x_2 > 2.5)$$

- If $x_2 > 2.5$, predicts $y = 1$.
- Otherwise, predicts $y = -1$

Row	x1	x2	y	Stump 1 Predicts
0	1	1	1	-1
1	1	3	1	1
2	2	2	-1	-1
3	2	4	1	1
4	3	1	-1	-1
5	3	2	-1	-1
6	3	5	1	1
7	4	4	1	1
8	5	2	-1	-1
9	5	5	-1	1

Error of the first stump

Row	x1	x2	y	Stump 1 Predicts	Weight 1	
0	1	1	1	-1	0.1	<-
1	1	3	1	1	0.1	
2	2	2	-1	-1	0.1	
3	2	4	1	1	0.1	
4	3	1	-1	-1	0.1	
5	3	2	-1	-1	0.1	
6	3	5	1	1	0.1	
7	4	4	1	1	0.1	
8	5	2	-1	-1	0.1	
9	5	5	-1	1	0.1	<-

Error of the first stump

- Stump 1 has 2 misclassifications at row 0 and 9 (The predictions are NOT the same as the y values). The total weights of these rows are:

$$\epsilon_1 = 0.1 + 0.1 = 0.2$$

Row	x1	x2	y	Stump 1 Predicts	Weight 1	
0	1	1	1	-1	0.1	<-
1	1	3	1	1	0.1	
2	2	2	-1	-1	0.1	
3	2	4	1	1	0.1	
4	3	1	-1	-1	0.1	
5	3	2	-1	-1	0.1	
6	3	5	1	1	0.1	
7	4	4	1	1	0.1	
8	5	2	-1	-1	0.1	
9	5	5	-1	1	0.1	<-

Voting Power of the first Stump

- Stump 1 has 2 misclassifications at row 0 and 9 (The predictions are NOT the same as the y values). The total weights of these rows are:

$$\epsilon_1 = 0.1 + 0.1 = 0.2$$

- Voting Power: (L is the learning rate. $L = 1$ in this example 1)

$$\alpha_1 = L \cdot \frac{1}{2} \cdot \ln(\frac{1-\epsilon_1}{\epsilon_1}) = 0.693$$

Row	x1	x2	y	Stump 1 Predicts	Weight 1	
0	1	1	1	-1	0.1	<-
1	1	3	1	1	0.1	
2	2	2	-1	-1	0.1	
3	2	4	1	1	0.1	
4	3	1	-1	-1	0.1	
5	3	2	-1	-1	0.1	
6	3	5	1	1	0.1	
7	4	4	1	1	0.1	
8	5	2	-1	-1	0.1	
9	5	5	-1	1	0.1	<-

Calculating the new weights

- For misclassified rows 0 and 9:

$$w_{new} = w_{old} \cdot e^{\alpha} = 0.1 \cdot e^{0.693} = 0.2$$

- For the correctly classified rows:

$$w_{new} = w_{old} \cdot e^{-\alpha} = 0.1 \cdot e^{-0.693} = .05$$

Row	x1	x2	y	Stump 1 Predicts	Weight 1	
0	1	1	1	-1	0.1	<-
1	1	3	1	1	0.1	
2	2	2	-1	-1	0.1	
3	2	4	1	1	0.1	
4	3	1	-1	-1	0.1	
5	3	2	-1	-1	0.1	
6	3	5	1	1	0.1	
7	4	4	1	1	0.1	
8	5	2	-1	-1	0.1	
9	5	5	-1	1	0.1	<-

Calculating the new weights

- For misclassified rows 0 and 9:

$$w_{new} = w_{old} \cdot e^{\alpha} = 0.1 \cdot e^{0.693} = 0.2$$

- For the correctly classified rows:

$$w_{new} = w_{old} \cdot e^{-\alpha} = 0.1 \cdot e^{-0.693} = .05$$

Row	**x1**	**x2**	**y**	**Stump 1 Predicts**	**Weight 1**	**Weight 2**
0	1	1	1	-1	0.1	0.2
1	1	3	1	1	0.1	0.05
2	2	2	-1	-1	0.1	0.05
3	2	4	1	1	0.1	0.05
4	3	1	-1	-1	0.1	0.05
5	3	2	-1	-1	0.1	0.05
6	3	5	1	1	0.1	0.05
7	4	4	1	1	0.1	0.05
8	5	2	-1	-1	0.1	0.05
9	5	5	-1	1	0.1	0.2

Calculating the new weights

- For misclassified rows 0 and 9:

$$w_{new} = w_{old} \cdot e^{\alpha} = 0.1 \cdot e^{0.693} = 0.2$$

- For the correctly classified rows:

$$w_{new} = w_{old} \cdot e^{-\alpha} = 0.1 \cdot e^{-0.693} = .05$$

- Notice how the weights increase for misclassified rows and decrease otherwise.

Row	x1	x2	y	Stump 1 Predicts	Weight 1	Weight 2
0	1	1	1	-1	0.1	0.2
1	1	3	1	1	0.1	0.05
2	2	2	-1	-1	0.1	0.05
3	2	4	1	1	0.1	0.05
4	3	1	-1	-1	0.1	0.05
5	3	2	-1	-1	0.1	0.05
6	3	5	1	1	0.1	0.05
7	4	4	1	1	0.1	0.05
8	5	2	-1	-1	0.1	0.05
9	5	5	-1	1	0.1	0.2

Calculating the new weights

- The total weights has to be 1. We divide the weights by the total $(.2 * 2 + .05 * 8 = .8)$ to achieve this.

Row	x1	x2	y	Stump 1 Predicts	Weight 1	Weight 2
0	1	1	1	-1	0.1	0.2
1	1	3	1	1	0.1	0.05
2	2	2	-1	-1	0.1	0.05
3	2	4	1	1	0.1	0.05
4	3	1	-1	-1	0.1	0.05
5	3	2	-1	-1	0.1	0.05
6	3	5	1	1	0.1	0.05
7	4	4	1	1	0.1	0.05
8	5	2	-1	-1	0.1	0.05
9	5	5	-1	1	0.1	0.2

Calculating the new weights

- The total weights has to be 1. We divide the weights by the total $(.2 * 2 + .05 * 8 = .8)$ to achieve this.
- Divide Weight 2 by 0.8

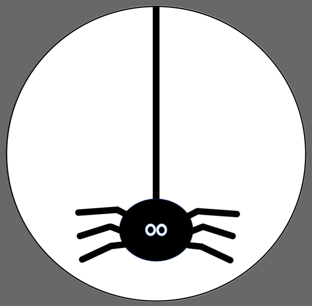
Row	x1	x2	y	Stump 1 Predicts	Weight 1	Weight 2
0	1	1	1	-1	0.1	0.2
1	1	3	1	1	0.1	0.05
2	2	2	-1	-1	0.1	0.05
3	2	4	1	1	0.1	0.05
4	3	1	-1	-1	0.1	0.05
5	3	2	-1	-1	0.1	0.05
6	3	5	1	1	0.1	0.05
7	4	4	1	1	0.1	0.05
8	5	2	-1	-1	0.1	0.05
9	5	5	-1	1	0.1	0.2

Calculating the new weights

- The total weights has to be 1. We divide the weights by the total $(.2 * 2 + .05 * 8 = .8)$ to achieve this.
- Divide Weight 2 by 0.8

Row	x1	x2	y	Stump 1 Predicts	Weight 1	Weight 2
0	1	1	1	-1	0.1	0.25
1	1	3	1	1	0.1	0.0625
2	2	2	-1	-1	0.1	0.0625
3	2	4	1	1	0.1	0.0625
4	3	1	-1	-1	0.1	0.0625
5	3	2	-1	-1	0.1	0.0625
6	3	5	1	1	0.1	0.0625
7	4	4	1	1	0.1	0.0625
8	5	2	-1	-1	0.1	0.0625
9	5	5	-1	1	0.1	0.25

Repeat the process to make the second Stump



Data to make the second Stump

Row	x1	x2	y	Weight 2
0	1	1	1	0.25
1	1	3	1	0.0625
2	2	2	-1	0.0625
3	2	4	1	0.0625
4	3	1	-1	0.0625
5	3	2	-1	0.0625
6	3	5	1	0.0625
7	4	4	1	0.0625
8	5	2	-1	0.0625
9	5	5	-1	0.25

Make the second stump

- Use Weighted Gini-Index to calculate the children entropy of all candidate splits

Row	x1	x2	y	Weight 2
0	1	1	1	0.25
1	1	3	1	0.0625
2	2	2	-1	0.0625
3	2	4	1	0.0625
4	3	1	-1	0.0625
5	3	2	-1	0.0625
6	3	5	1	0.0625
7	4	4	1	0.0625
8	5	2	-1	0.0625
9	5	5	-1	0.25

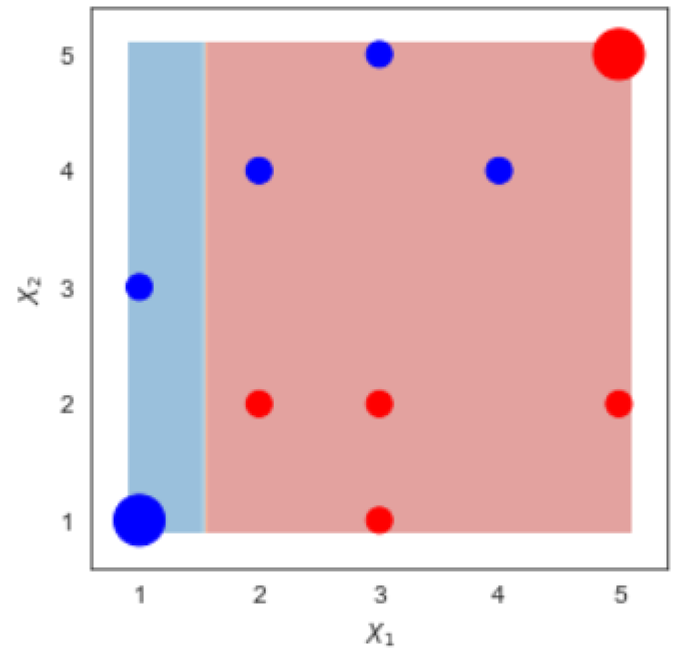
Make the second stump

- Use Weighted Gini-Index to calculate the children entropy of all candidate splits
- The split with the lowest children impurity is the best split

Row	x1	x2	y	Weight 2
0	1	1	1	0.25
1	1	3	1	0.0625
2	2	2	-1	0.0625
3	2	4	1	0.0625
4	3	1	-1	0.0625
5	3	2	-1	0.0625
6	3	5	1	0.0625
7	4	4	1	0.0625
8	5	2	-1	0.0625
9	5	5	-1	0.25

Make the second stump

- Use Weighted Gini-Index to calculate the children entropy of all candidate splits
- The split with the lowest children impurity is the best split



$\boxed{x_1 < 1.5}$ $= I(x_1 < 1.5)$

Y N

$\hat{y} = 1$ $\hat{y} = -1$

Error of the second stump

Row	x1	x2	y	Stump 2 Predicts	Weight 2	
0	1	1	1	1	0.25	
1	1	3	1	1	0.0625	
2	2	2	-1	-1	0.0625	
3	2	4	1	-1	0.0625	<-
4	3	1	-1	-1	0.0625	
5	3	2	-1	-1	0.0625	
6	3	5	1	-1	0.0625	<-
7	4	4	1	-1	0.0625	<-
8	5	2	-1	-1	0.0625	
9	5	5	-1	-1	0.25	

Error of the second stump

- Stump 2 has misclassifications at row 3, 6, and 7 (The predictions are NOT the same as the y values). The total weights of these rows are: 0.0625 + 0.0625 + 0.0625 = 0.1875

- Error of Stump 2:

$$\epsilon_2 = 0.1875$$

- Voting Power:

$$\alpha_2 = L \cdot \frac{1}{2} \cdot \ln(\frac{1 - \epsilon_2}{\epsilon_2}) = 0.733$$

Row	x1	x2	y	Stump 2 Predicts	Weight 2	
0	1	1	1	1	0.25	
1	1	3	1	1	0.0625	
2	2	2	-1	-1	0.0625	
3	2	4	1	-1	0.0625	<-
4	3	1	-1	-1	0.0625	
5	3	2	-1	-1	0.0625	
6	3	5	1	-1	0.0625	<-
7	4	4	1	-1	0.0625	<-
8	5	2	-1	-1	0.0625	
9	5	5	-1	-1	0.25	

Calculating the new weights

- For misclassified rows 3, 6 and 7:

$$w_{new} = w_{old} \cdot e^{\alpha}$$

- For the correctly classified rows:

$$w_{new} = w_{old} \cdot e^{-\alpha}$$

voting power

Row	x1	x2	y	Stump 2 Predicts	Weight 2	
0	1	1	1	1	0.25	
1	1	3	1	1	0.0625	
2	2	2	-1	-1	0.0625	
3	2	4	1	-1	0.0625	<-
4	3	1	-1	-1	0.0625	
5	3	2	-1	-1	0.0625	
6	3	5	1	-1	0.0625	<-
7	4	4	1	-1	0.0625	<-
8	5	2	-1	-1	0.0625	
9	5	5	-1	-1	0.25	

Calculating the new weights

- For misclassified rows 3, 6 and 7:

$$w_{new} = w_{old} \cdot e^{\alpha}$$

- For the correctly classified rows:

$$w_{new} = w_{old} \cdot e^{-\alpha}$$

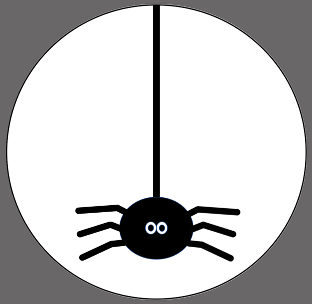
Row	x1	x2	y	Weight 2	Stump 2 Predicts	Weight 3
0	1	1	1	0.25	1	0.12012
1	1	3	1	0.0625	1	0.03003
2	2	2	-1	0.0625	-1	0.03003
3	2	4	1	0.0625	-1	0.13008
4	3	1	-1	0.0625	-1	0.03003
5	3	2	-1	0.0625	-1	0.03003
6	3	5	1	0.0625	-1	0.13008
7	4	4	1	0.0625	-1	0.13008
8	5	2	-1	0.0625	-1	0.03003
9	5	5	-1	0.25	-1	0.12012

Normalize the new weights

- The total weights has to be 1. We divide Weight 3 by the total of current Weight 3, which is 0.780624761 to achieve this.

Row	x1	x2	y	Weight 2	Stump 2 Predicts	Weight 3
0	1	1	1	0.25	1	0.15387
1	1	3	1	0.0625	1	0.03847
2	2	2	-1	0.0625	-1	0.03847
3	2	4	1	0.0625	-1	0.16664
4	3	1	-1	0.0625	-1	0.03847
5	3	2	-1	0.0625	-1	0.03847
6	3	5	1	0.0625	-1	0.16664
7	4	4	1	0.0625	-1	0.16664
8	5	2	-1	0.0625	-1	0.03847
9	5	5	-1	0.25	-1	0.15387

Repeat the process to make the third Stump



Data to Make the third stump

Row	x1	x2	y	Weight 3
0	1	1	1	0.15387
1	1	3	1	0.03847
2	2	2	-1	0.03847
3	2	4	1	0.16664
4	3	1	-1	0.03847
5	3	2	-1	0.03847
6	3	5	1	0.16664
7	4	4	1	0.16664
8	5	2	-1	0.03847
9	5	5	-1	0.15387

Make the third stump

- Use Weighted Gini-Index to calculate the children entropy of all candidate splits

Row	x1	x2	y	Weight 3
0	1	1	1	0.15387
1	1	3	1	0.03847
2	2	2	-1	0.03847
3	2	4	1	0.16664
4	3	1	-1	0.03847
5	3	2	-1	0.03847
6	3	5	1	0.16664
7	4	4	1	0.16664
8	5	2	-1	0.03847
9	5	5	-1	0.15387

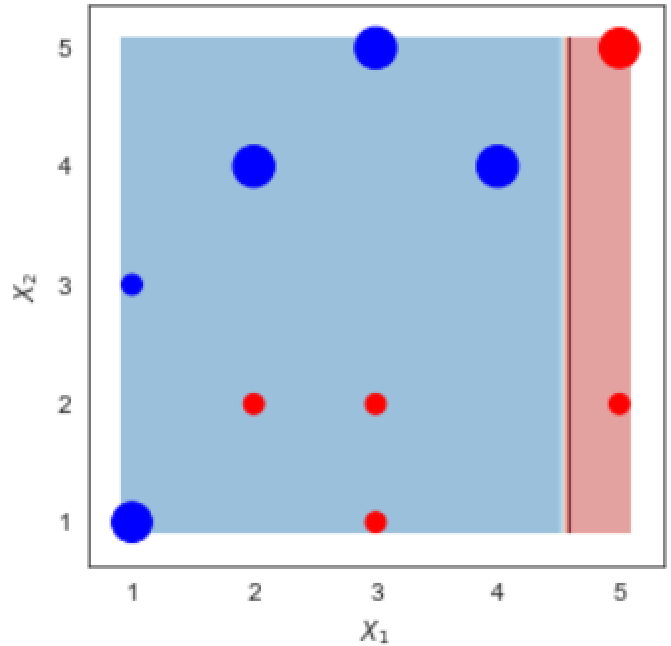
Make the third stump

- Use Weighted Gini-Index to calculate the children entropy of all candidate splits
- The split with the lowest children impurity is the best split

Row	x1	x2	y	Weight 3
0	1	1	1	0.15387
1	1	3	1	0.03847
2	2	2	-1	0.03847
3	2	4	1	0.16664
4	3	1	-1	0.03847
5	3	2	-1	0.03847
6	3	5	1	0.16664
7	4	4	1	0.16664
8	5	2	-1	0.03847
9	5	5	-1	0.15387

Make the third stump

- Use Weighted Gini-Index to calculate the children entropy of all candidate splits
- The split with the lowest children impurity is the best split



$I(x_1 < 4.5)$

Error of the third stump

Row	x1	x2	y	Stump 3 Predicts	Weight 3	
0	1	1	1	1	0.15385	
1	1	3	1	1	0.03846	
2	2	2	-1	1	0.03846	<-
3	2	4	1	1	0.16667	
4	3	1	-1	1	0.03846	<-
5	3	2	-1	1	0.03846	<-
6	3	5	1	1	0.16667	
7	4	4	1	1	0.16667	
8	5	2	-1	-1	0.03846	
9	5	5	-1	-1	0.15385	

Error of the third stump

- Stump 3 has misclassifications at row 2, 4, and 5 (The predictions are NOT the same as the y values). The total weights of these rows are:

$$\epsilon_3 = 0.03846 \cdot 3 = 0.11538$$

- Voting Power:

$$\alpha_3 = L \cdot \frac{1}{2} \cdot \ln(\frac{1-\epsilon_3}{\epsilon_3}) = 1.018$$

Learning rate

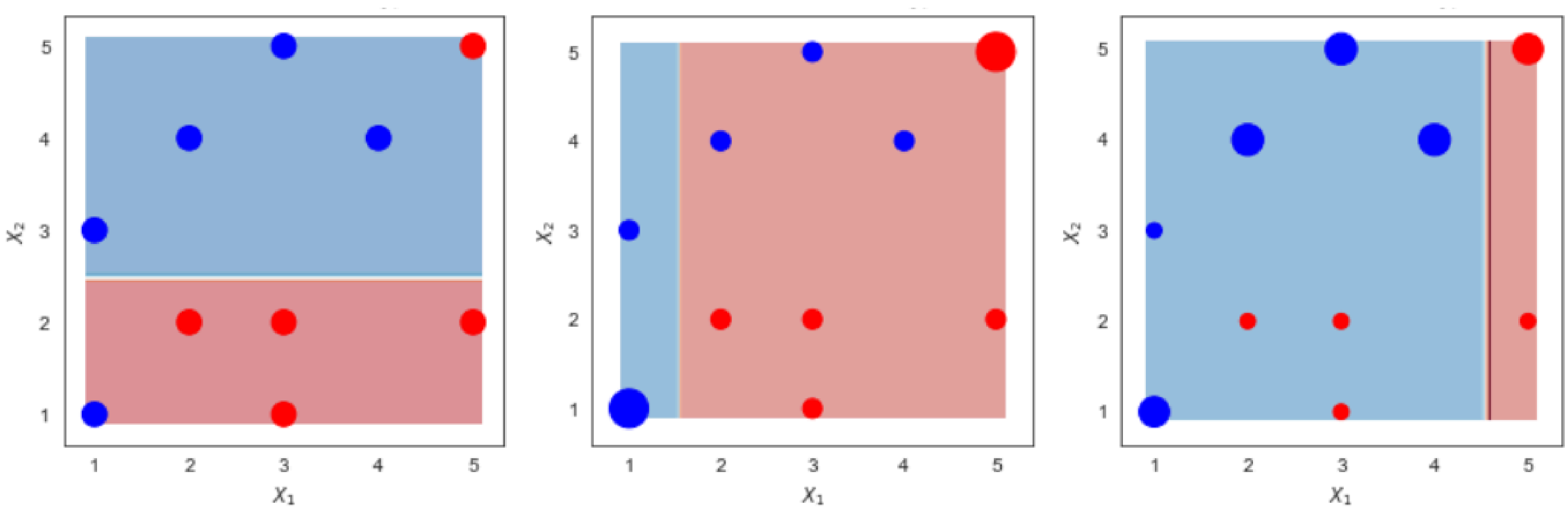
Row	x1	x2	y	Stump 3 Predicts	Weight 3	
0	1	1	1	1	0.15385	
1	1	3	1	1	0.03846	
2	2	2	-1	1	0.03846	<-
3	2	4	1	1	0.16667	
4	3	1	-1	1	0.03846	<-
5	3	2	-1	1	0.03846	<-
6	3	5	1	1	0.16667	
7	4	4	1	1	0.16667	
8	5	2	-1	-1	0.03846	
9	5	5	-1	-1	0.15385	

Summarise the results

Row	x1	x2	y	Stump 1 Predicts	Weight 1	Weight 2	Stump 2 Predicts	Weight 3	Stump 3 Predicts
0	1	1	1	-1	0.1	0.25	1	0.153846	1
1	1	3	1	1	0.1	0.0625	1	0.0384615	1
2	2	2	-1	-1	0.1	0.0625	-1	0.0384615	1
3	2	4	1	1	0.1	0.0625	-1	0.166667	1
4	3	1	-1	-1	0.1	0.0625	-1	0.0384615	1
5	3	2	-1	-1	0.1	0.0625	-1	0.0384615	1
6	3	5	1	1	0.1	0.0625	-1	0.166667	1
7	4	4	1	1	0.1	0.0625	-1	0.166667	1
8	5	2	-1	-1	0.1	0.0625	-1	0.0384615	-1
9	5	5	-1	1	0.1	0.25	-1	0.153846	-1

Combining three Stumps

- Let say we stop making new stumps here.
- We will combine the three stumps to make the final model



$\alpha_1 = .693$ $\qquad$ $\alpha_2 = .733$ $\qquad$ $\alpha_3 = 1.018$

$I_1 = I(X_2 > 2.5)$ $\qquad$ $I_2 = I(X_1 < 1.5)$ $\qquad$ $I_3 = I(X_1 < 4.5)$

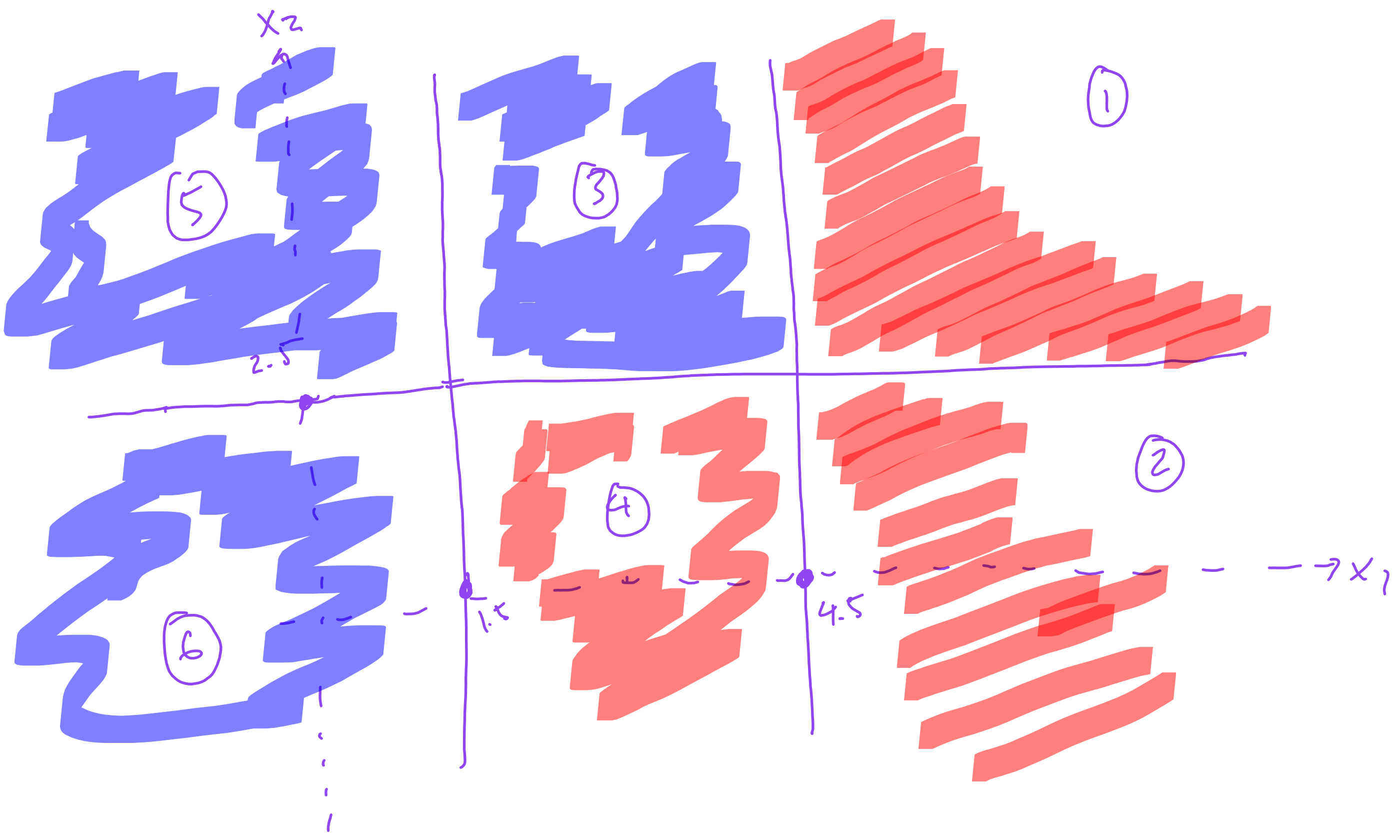
$$\text{Final } \hat{y} = \text{sign}\left(\alpha_1 I_1 + \alpha_2 I_2 + \alpha_3 I_3 \right)$$

$$\text{Final } \hat{y} = \text{sign}\left(\alpha_1 I_1 + \alpha_2 I_2 + \alpha_3 I_3\right)$$

Note

$$\text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

$$\hat{y} = \text{sign}\left[.693 \cdot I(x_2 > 2.5) + .733\, I(x_1 < 1.5) + 1.018\, I(x_1 < 4.5) \right]$$

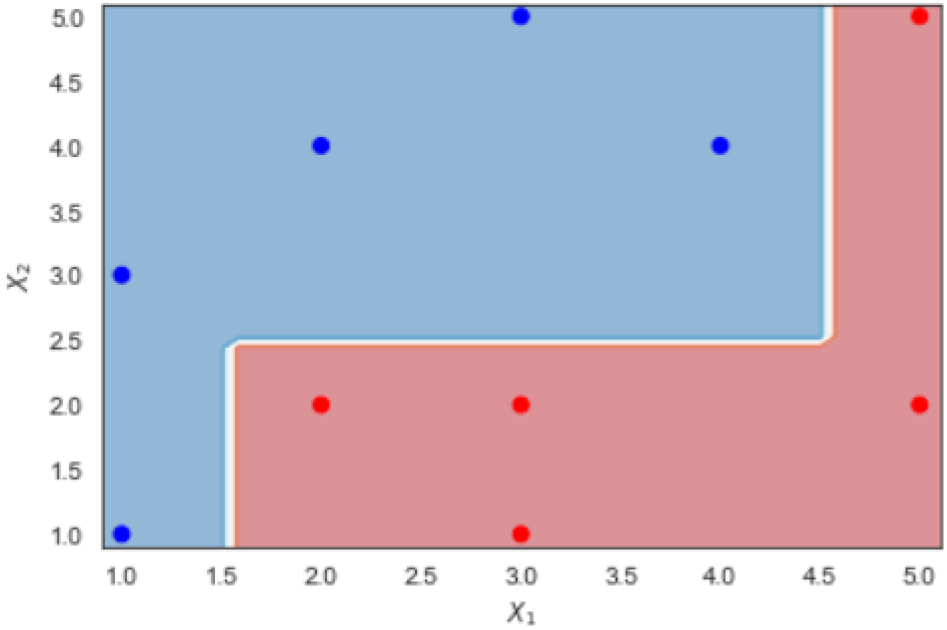


For 1 $\quad \underline{x_2 > 2.5}, \quad \underline{x_1 > 1.5}, \quad \underline{x_1 > 4.5}$

$$\text{sign}\left[.693 \cdot \underbrace{I(x_2 > 2.5)}_{=1} + .733\, \underbrace{I(x_1 < 1.5)}_{-1} + 1.018\, \underbrace{I(x_1 < 4.5)}_{-1} \right]$$

$$\text{sign}\left[.693 * 1 - .733 * 1 - 1.018 * 1 \right] = \text{sign}(-1.058) = -1$$

Combining three Stumps



Learning rate